CHAPTER

10 Hybrid Parameters

10.1. INTRODUCTION

The basic construction, appearance and characteristics of bipolar junction transistors (BJTs) were discussed in chapter 8. We now begin to examine the *small-signal* ac response of the BJT amplifier by reviewing the *models* most frequently employed for representing the transistor in the sinusoidal ac domain.

One of our first concerns in the sinusoidal ac analysis of transistor networks is the magnitude of the input signal (an amplifier may use either small signal operation or large signal operation). Study of small signal operation may be done either graphically or by using small signal equivalent circuit for the BJT operating in the active region, however, the second method is more convenient. The large signal operation may be best studied graphically because of involvement of certain non-linear operation in it. For study of small signal operation, the transistor may be replaced by its equivalent circuit (or model) and then the usual method of network analysis may be used to obtain expressions for its operating characteristics *e.g.*, input impedance, output impedance, voltage gain, current gain etc.

We here take up the study of hybrid (h-) parameter equivalent circuit (or model) for the transistor and analysis of low frequency small signal common-emitter (CE) amplifier using this model.

10.2. TWO-PORT DEVICES AND THE HYBRID MODEL

A box representing a two-port network is illustrated in Fig. 10.1. The terminal behaviour of a two-port device may be specified by two voltages and two currents (voltage v_1, and current i_1 at the input port and voltage v_2 and current i_2 at the output port). The conventional positive polarities* of voltages v_1 and v_2 and currents i_1 and i_2 are shown in the figure. Out of four quantities (v_1, v_2, i_1 and i_2), any two may be selected as independent variables** and the remaining two be expressed in terms of the selected independent variables. This leads to various two-port parameters, out of which following three are more important.

1. Open-circuit impedance parameters or Z-parameters Z_{11}, Z_{12}, Z_{21} and Z_{22}
2. Short-circuit admittance parameters or Y-parameters Y_{11}, Y_{12}, Y_{21} and Y_{22}
3. Hybrid parameters or the h-parameters.

Fig. 10.1. ***Conventional Positive Polarities of Voltages and Currents in a Two-Port Network***

In transistor amplifier analysis, Z- and Y-parameters were used earlier. But now hybrid or the h-parameters alone are used in a transistor circuit analysis and, therefore, only the h-parameters will be taken here for discussion.

10.2.1. Hybrid Parameters or h-Parameters.

For the two-port network illustrated in Fig. 10.1, if input current i_1 and the output voltage v_2 are taken as independent variables and the two-port shown in the figure is linear, we may write

$$v_1 = h_{11}\, i_1 + h_{12}\, v_2 \qquad \text{...(10.1)}$$

$$i_2 = h_{21}\, i_1 + h_{22}\, v_2 \qquad \text{...(10.2)}$$

In the above equations, the *hs* are fixed for a given circuit and are called the hybrid or h-parameters. Because these four parameters have mixed dimensions (h_{11} has dimension of ohm, h_{12} and h_{21} are dimensionless, and h_{22} has dimension of mho or siemen) so they are called hybrid or h-parameters.

1. ***Meaning of h-parameters.*** By assuming that the given two-port network has no reactive element and by applying open-circuit ($i_1 = 0$) or short-circuit ($v_2 = 0$) conditions to Eqs. (10.1) and (10.2), the h-parameters can be defined as below :

If the output terminals are short-circuited, (Fig. 10.2) output voltage v_2 becomes zero and Eqs. (10.1) and (10.2) become

$$v_1 = h_{11}\, i_1 + h_{12} \times 0 = h_{11}\, i_1$$

and $$i_2 = h_{21}\, i_1 + h_{22} \times 0 = h_{21}\, i_1$$

or $$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2 = 0}$$

and $$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2 = 0}$$

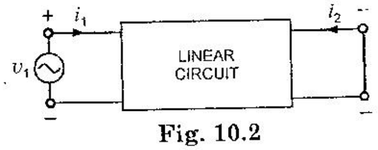


Fig. 10.2

* The currents are taken positive when they enter the device and negative when they leave it.

** In general, we are not free to select the independent variables arbitrarily—for example in case of an ideal transformer, two voltages v_1 and v_2 cannot be selected as the independent variables because their ratio is a constant and is equal to the turn-ratio of the transformer.

Since h_{11} is the ratio of input voltage and input current with output terminals short-circuited, it is called the *input impedance with output short-circuited.* The subscript 11 of h_{11} defines the fact that the parameter is determined by the ratio of quantities measured at the input terminals. Its unit is ohm.

Similarly h_{21} is the ratio of output and input currents (*i.e.* i_2/i_1) with output terminals short-circuited, so it is called the *forward transfer current gain with output short-circuited.* Obviously it is dimensionless quantity.

If the input terminals are open-circuited and we drive the output terminals with voltage v_2, as shown in Fig. 10.3, input current i_1 becomes zero and Eqs. (10.1) and (10.2) become

$$v_1 = h_{11} \times 0 + h_{12}\, v_2 = h_{12}\, v_2$$

and $$i_2 = h_{21} \times 0 + h_{22}\, v_2 = h_{22}\, v_2$$

or $$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1 = 0}$$

and $$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1 = 0}$$

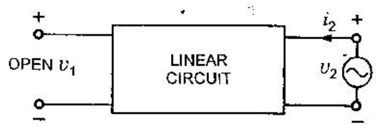


Fig. 10.3

Thus the parameter h_{12} is the ratio of input voltage to the output voltage with zero input current (*i.e.* $i_1 = 0$). It is dimensionless quantity and is called the open-circuit *reverse transfer voltage ratio*, the subscript 12 of h_{12} reveals that the parameter is a transfer quantity determined by the ratio of input to output measurements.

Similarly h_{22} is the ratio of output current to the output voltage with zero input current (*i.e.* $i_1 = 0$). It is called the *open-circuit output admittance* and is measured in siemens. The subscript 22 in h_{22} indicates that it is determined by a ratio of output quantities.

2. *Notations.* The convenient alternative subscript notations recommended by the IEEE Standards are given below :

i = 11 = input $\quad$ o = 22 = output

f = 21 = forward transfer $\quad$ r = 12 = reverse transfer

In case of transistors, another subscript (b, e or c) is added to designate the type of transistor configuration. For example $h_{ie} = h_{11e}$ = input resistance in common emitter configuration.

Notations used in transistor amplifier for the three configurations are tabulated below (Table 10.1).

Since the two-port network (or the device) described by Eqs. (10.1) and (10.2) is assumed to have no reactive elements, the four parameters h_{11}, h_{12}, h_{21} and h_{22} are real numbers and voltages and currents v_1, v_2, and i_1, i_2 are functions of time. However, if the reactive elements had been included in the device, the excitation would be considered to be sinusoidal, the h-parameters would in general be functions of frequency, and the voltages and currents would be represented by phasors V_1, V_2, and I_1, I_2.

10.2.2. Hybrid Model. The hybrid circuit for any two-port network characterized by Eqs. (10.1) and (10.2) is shown in Fig. 10.4. If Kirchhoff's voltage

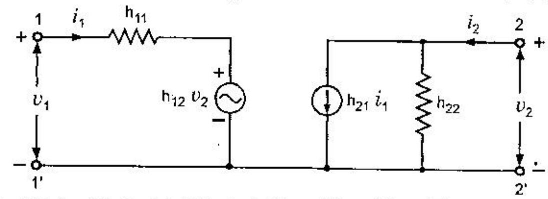


Fig. 10.4. *Hybrid Model For The Two Port Network Shown in Fig. 10.1*

law and Kirchhoff's current law are applied to the input and output ports, Eqs. (10.1) and (10.2) respectively will be obtained. Thus model given in Fig. 10.4 truely satisfies Eqs. (10.1) and (10.2).

The input circuit derived from Eq. (10.1) appears as a resistance h_{11} in series with a voltage generator $h_{12}\, v_2$. The output circuit, derived from Eq. (10.2) consists of a current generator $h_{21} i_1$ and shunt resistance h_{22}. This circuit is called *hybrid equivalent* because its input portion is a Thevenin's equivalent (or a voltage generator in series with a resistance) while the output portion is a Norton's equivalent (or a current generator with shunt resistance). Thus it is a mixture or hybrid. The symbol '*h*' is simply the abbreviation of the word hybrid (hybrid means "mixed").

The hybrid equivalent circuit (or model) given in Fig. 10.4 is an extremely important one in the area of electronics today. It will appear over and over again in the analysis to follow. There are two main reasons of popularity of hybrid model. First, it isolates the input and output circuits, their interaction being accounted for by the two controlled voltage and current sources—the effect of output upon input is represented by the equivalent voltage generator $h_{12} v_2$ and the effect of input upon output is represented by the current generator $h_{21} i_1$. The value of the former depends upon the output voltage v_2 while the value for the latter depends upon the input current i_1. Secondly, the two portions of the circuit are in a form which makes it simple to take into account the source and the load circuits.

TABLE 10.1

S.No.	*h-parameter*	*Common Base Configuration*	*Common Emitter Configuration*	*Common Collector Configuration*
1.	h_{11}	h_{ib}	h_{ie}	h_{ic}
2.	h_{12}	h_{rb}	h_{re}	h_{rc}
3.	h_{21}	h_{fb}	h_{fe}	h_{fc}
4.	h_{22}	h_{ob}	h_{oe}	h_{oc}

10.3. TRANSISTOR HYBRID MODEL

The basic assumption in arriving at a transistor linear model or equivalent circuit is that the variations about the operating or quiescent point are small and, therefore, the transistor parameters can be considered constant over the small range of operation.

Many transistor models have been proposed, each one having its particular merits and demerits. The transistor model presented here, is given in terms of the h-parameters, which are real numbers at audio-frequencies, are easy to measure, can also be obtained from the static characteristics of a transistor, and are particularly convenient to use in analysis and design of circuit. Furthermore, a set of h-parameters is specified for many transistors by the manufacturers.

To derive a hybrid model for a transistor, let us consider the basic CE amplifier circuit given in Fig. 10.7. The variables i_B, i_C, v_B and v_C represent the total instantaneous values of currents and voltages. We may select the input current i_B and output voltage v_C as independent variables. Since input voltage v_B is some function f_1 of i_B and v_C and output current i_C is another function f_2 of i_B and v_C, we may write

$$v_B = f_1\,(i_B, v_C) \quad \text{...(10.3)}$$

$$i_C = f_2\,(i_B, v_C) \quad \text{...(10.4)}$$

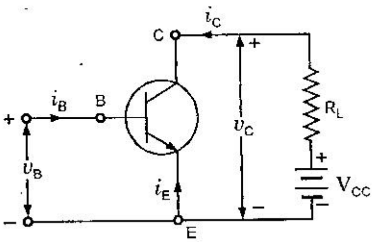


Fig. 10.7. *Basic CE Configuration*

Making a Taylor's series expansion of Eqs. (10.3) and (10.4) about the zero signal operating point (I_B, V_C) and neglecting higher order terms we have

$$\Delta v_B = \left.\frac{\delta f_1}{\delta i_B}\right|_{V_C} \Delta i_B + \left.\frac{\delta f_1}{\delta v_C}\right|_{I_B} \Delta v_C \quad \text{...(10.5)}$$

$$\Delta i_C = \left.\frac{\delta f_2}{\delta i_B}\right|_{V_C} \Delta i_B + \left.\frac{\delta f_2}{\delta v_C}\right|_{I_B} \Delta v_C \quad \text{...(10.6)}$$

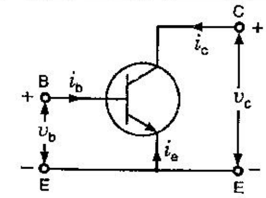
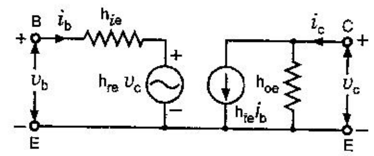
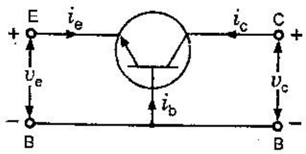
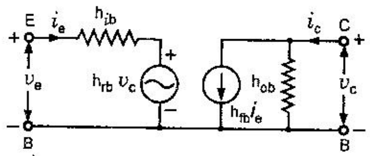
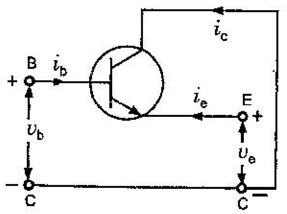
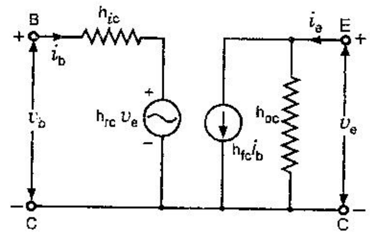
where partial derivatives $\frac{\delta f_1}{\delta i_B}$ and $\frac{\delta f_2}{\delta i_B}$ are taken keeping collector voltage V_C constant while partial derivates $\frac{\delta f_1}{\delta v_C}$ and $\frac{\delta f_2}{\delta v_C}$ are taken keeping base current I_B constant.

The quantities Δv_B, Δv_C, Δi_B and Δi_C represent the small-signal (incremental) base and collector voltages and currents and may be represented as v_b, v_c, i_b, and i_c respectively as per standard notations. We may now write Eqs. (10.5) and (10.6) as below

$$v_b = h_{ie}\, i_b + h_{re}\, v_c \quad \text{...(10.7)}$$

$$i_c = h_{fe}\, i_b + h_{oe}\, v_c \quad \text{...(10.8)}$$

TABLE 10.2

Configuration	*Circuit Schematic*	*Hybrid Model v-i*	*Equations*
Common Emitter, CE			$v_b = h_{ie}\, i_b + h_{re}\, v_c$ $i_c = h_{fe} i_b + h_{oe}\, v_c$
Common Base, CB			$v_e = h_{ib}\, i_e + h_{rb}\, v_c$ $i_c = h_{fb}\, i_e + h_{ob}\, v_c$
Common Collector CC			$v_b = h_{ic}\, i_b + h_{rc}\, v_e$ $i_e = h_{fc}\, i_b + h_{oc}\, v_e$

where $h_{ie} = \left.\dfrac{\delta f_1}{\delta i_B}\right|_{V_C} = \left.\dfrac{\delta v_B}{\delta i_B}\right|_{V_C} = \left.\dfrac{v_b}{i_b}\right|_{V_C = 0}$ ...(10.9*a*)

$$h_{fe} = \left.\frac{\delta f_2}{\delta i_B}\right|_{V_C} = \left.\frac{\delta i_C}{\delta i_B}\right|_{V_C} = \left.\frac{i_c}{i_b}\right|_{V_C = 0} \quad ...(10.9b)$$

$$h_{re} = \left.\frac{\delta f_1}{\delta v_C}\right|_{I_B} = \left.\frac{\delta v_B}{\delta v_C}\right|_{I_B} = \left.\frac{v_b}{v_c}\right|_{I_B = 0} \quad ...(10.9c)$$

$$h_{oe} = \left.\frac{\delta f_2}{\delta v_C}\right|_{I_B} = \left.\frac{\delta i_C}{\delta v_C}\right|_{I_B} = \left.\frac{i_c}{v_c}\right|_{I_B = 0} \quad ...(10.9d)$$

The partial derivatives of Eqs. (10.9) define the h-parameters for the transistor in common-emitter (CE) configuration.

Equations (10.7) and (10.8) are found to be of exactly the same form as Eqs. (10.1) and (10.2) hence, the model shown in Fig. 10.4 can be used to represent a transistor.

The common-emitter (CE), common-base (CB), and common-collector (CC) configurations, their hybrid models and their terminal volt-ampere equations are summarized in Table 10.2.

The circuits and equations in above Table 10.2 are valid for either an N-P-N or P-N-P transistor and are independent of the type of load or biasing method.

10.4. EXPERIMENTAL DETERMINATION OF HYBRID PARAMETERS

Determination of hybrid parameters of a general linear circuit has already been discussed in Art. 10.2.1. For determination of hybrid parameters for a CE transistor, consider the circuits given in Fig. 10.8. The rms values will be considered in discussion. Volt-ampere equations for a CE transistor are as given below :

$$V_{be} = h_{ie}\, I_b + h_{re}\, V_{ce} \quad ...(10.10)$$

$$I_c = h_{fe}\, I_b + h_{oe}\, V_{ce} \quad ...(10.11)$$

For measurement of h_{ie} and h_{fe} parameters, output is ac short-circuited [Fig. 10.8 (*a*)] by making capacitance C_2 deliberately large. The result is that the changing component of collector current flows through C_2 instead of R_C and ac voltage developed across C_2 is zero *i.e.* $V_{ce} = 0$. Here it should be clarified that setting $V_{ce} = 0$ does not mean that the dc collector-emitter voltage, V_{CE} is zero. It simply means that ac output is short-circuited.

Substituting $V_{ce} = 0$ in Eqs. (10.10) and (10.11) we have

$$V_{be} = h_{ie}\, I_b + h_{re} \times 0$$

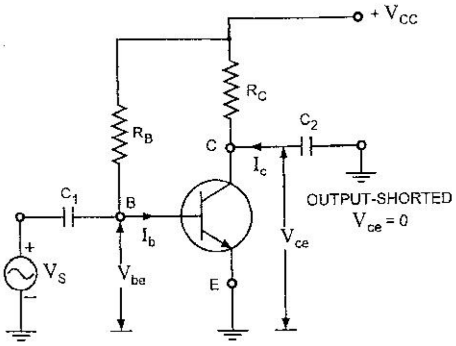
$$\text{or} \quad h_{ie} = \frac{V_{be}}{I_b} \text{ for } V_{ce} = 0 \quad ...(10.12)$$

$$\text{and} \quad I_c = h_{fe}\, I_b + h_{oe} \times 0$$

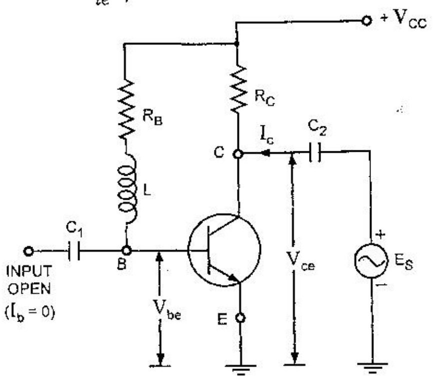
$$\text{or} \quad h_{fe} = \frac{I_c}{I_b} \text{ for } V_{ce} = 0 \quad ...(10.13)$$

Thus from Eqs. (10.12) and (10.13) h-parameters h_{ie} and h_{fe} can be determined. It is to be noted here that I_b and I_c are ac rms base and collector currents respectively. Also V_{be} is the ac rms value of base-emitter voltage.

For measuring hybrid parameters h_{oe} and h_{re} the input is ac open-circuited, a signal generator is applied across the output and a large inductor L is connected in series with R_B, as illustrated in Fig. 10.8 (*b*). The inductor L, having dc resistance very small, does not disturb the operating point but does not allow flow of ac current through R_B. Furthermore, a voltmeter of high input impedance is used to measure base-emitter voltage V_{be} and hence there are no paths connected to the base with any appreciable ac current. Thus base is effectively ac open-circuited *i.e.* $I_b = 0$.



(a) Circuit For Measuring h_{fe} and h_{ie} of CE Transistor



(b) Circuit For Measuring h_{re} and h_{oe} of CE Transistor

Fig. 10.8

Substituting $I_b = 0$ in Eqs. (10.10) and (10.11) we have

$$V_{be} = h_{ie} \times 0 + h_{re} V_{ce}$$

$$\text{or} \quad h_{re} = \frac{V_{be}}{V_{ce}} \text{ for } I_b = 0 \qquad \text{...(10.14)}$$

$$\text{and } I_c = h_{fe} \times 0 + h_{oe} V_{ce}$$

$$\text{or } h_{oe} = \frac{I_c}{V_{ce}} \text{ for } I_b = 0 \qquad \text{...(10.15)}$$

Thus by measurement of V_{be}, V_{ce} and I_c, the hybrid parameters h_{re} and h_{oe} can be determined.

Example 10.2. The following test results were obtained in a CE amplifier circuit while measuring h-parameters experimentally.

(*i*) With ac output shorted $I_b = 20\ \mu A$, $I_c = 1$ mA, $V_{be} = 22$ mV and $V_{ce} = 0$

(*ii*) With ac input open-circuited $I_b = 0$, $V_{be} = 0.25$ mV, $I_c = 30\ \mu A$ and $V_{ce} = 1$ V

Determine hybrid parameters of the given transistor.

Solution : *From short-circuit test data*

$$h_{ie} = \frac{V_{be}}{I_b} \quad (\text{for } V_{ce} = 0) = \frac{22 \times 10^{-3}}{20 \times 10^{-6}} = 1.1\ k\Omega \textbf{ Ans.}$$

refer to Eq. (10.12)

$$h_{fe} = \frac{I_c}{I_b} \quad (\text{for } V_{ce} = 0) = \frac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50 \textbf{ Ans.}$$

refer to Eq. (10.13)

From open-circuit test data

$$h_{re} = \frac{V_{be}}{V_{ce}} \ (\text{for } I_b = 0) = \frac{0.25 \times 10^{-3}}{1} = 2.5 \times 10^{-4} \textbf{ Ans.}$$

refer to Eq. (10.14)

$$h_{oe} = \frac{I_c}{V_{ce}} \ (\text{for } I_b = 0) \qquad \text{refer to Eq. (10.15)}$$

$$= \frac{30 \times 10^{-6}}{1} = 30 \times 10^{-6} \text{ S or } 30\ \mu S \textbf{ Ans.}$$

10.5. DETERMINATION OF h-PARAMETERS FROM STATIC CHARACTERISTICS

Functional relationships for the CE configuration of total instantaneous collector current and base voltage in terms of two variables (base current and collector voltage) are given by Eqs. (10.4) and (10.3) respectively. Such functional relationships are represented by families of characteristic curves of transistors, already described in chapter 8. Two families of curves are usually specified for transistors. The output characteristic curves depict the relationship between the output current and voltage, with input current as the parameter. Figures. 8.18, 8.25 and 8.29 give the typical output characteristic curves for the common-base, common-emitter and common-collector configurations of a transistor. The input characteristic curves give the relationship between input voltage and current with output voltage as the parameter. Typical input characteristic curves for the common-base, common-emitter and common-collector configurations of a transistor are illustrated in Figs. 8.17, 8.24 and 8.28 respectively. h-parameters can be determined graphically from the input and output characteristics of a transistor for a particular configuration.

10.5.1. Determination of Hybrid Parameters h_{fe} and h_{oe}.

For a common-emitter configuration, the output characteristic curves given in Fig. 8.25 are reproduced in Fig. 10.9. In this figure let us consider the curve for $i_B = I_B = 60\ \mu A$. At a point Q on this curve, the quiescent collector current and collector voltage are I_C and V_C respectively. If a vertical straight line is drawn through point Q intersecting curves for $i_{B_1} = 40\ \mu A$ and $i_{B_2} = 80\ \mu A$ at points Q_1 and Q_2 respectively, the corresponding collector currents will be i_{C_1} and i_{C_2} respectively,

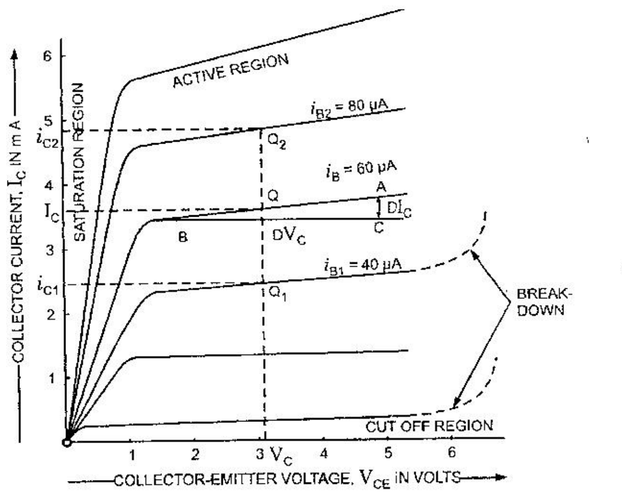


Fig. 10.9. ***Output Characteristics For Common Emitter N-P-N Transistor***

Now from Eqs. [10.9 (*b*)] and [10.9 (*d*)] we have

$$h_{fe} = \frac{\delta i_C}{\delta i_B} = \left.\frac{\Delta i_C}{\Delta i_B}\right|_{V_C} = \frac{i_{C_2} - i_{C_1}}{i_{B_2} - i_{B_1}} \qquad ...(10.16)$$

and $$h_{oe} = \frac{\delta i_C}{\delta v_C} = \left.\frac{\Delta i_C}{\Delta v_C}\right|_{I_B}$$

= Slope of the output characteristic curve at the point

$$= \frac{AC}{BC} \qquad ...(10.17)$$

Thus the value of h_{oe} at point Q is given by the slope of the output characteristic curve at that point. Thus slope can be determined by drawing a line tangential to the characteristic curve at the point Q. The slope can also be determined by drawing an incremental triangle ABC about point Q and noting the values of AC and BC.

The parameter h_{fe} is the most important transistor small signal parameter. This common-emitter current transfer ratio, or CE alpha, is also written α_e or β', and called the *small-signal beta* of the transistor.

10.5.2. Determination of Hybrid Parameters h_{re} and h_{ie}. For a common-emitter configuration, the input characteristics are shown in Fig. 10.10. In this figure let us consider the curve for $v_C = V_C$. At a point Q on this curve, the quiescent base voltage and base current are V_B and I_B respectively. If a vertical straight line is drawn through this point Q intersecting curves for v_{C_1} and v_{C_2} at points Q_1 and Q_2 respectively, the corresponding values of base voltages will be v_{B_1} and v_{B_2} respectively.

Now from Eqs. 10.9 (*c*) and 10.9 (*a*) we have

$$h_{re} = \frac{\delta v_B}{\delta v_C} = \left.\frac{\Delta v_B}{\Delta v_C}\right|_{I_B} = \frac{v_{B_2} - v_{B_1}}{v_{C_2} - v_{C_1}} \qquad ...(10.18)$$

$$h_{ie} = \frac{\delta v_B}{\delta i_B} = \left.\frac{\Delta v_B}{\Delta i_B}\right|_{V_C}$$ = Slope of the input characteristic curve at the point $= \frac{AC}{BC}$ ...(10.19)

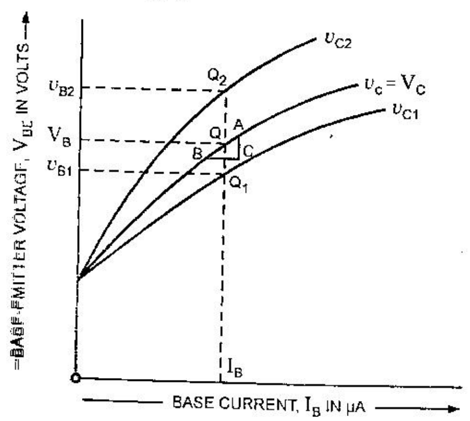


Fig. 10.10. *Input Characteristics For Common Emitter N-P-N Transistor*

Since h_{re} is of the order of 10^{-4} so $\Delta v_B << \Delta v_C$ and hence the above method is not accurate in practice, though it is correct in principle.

Slope of the input characteristic curve giving the value of h_{ie} can be determined by either drawing a straight tangential line to the input characteristic curve at point Q or by drawing an incremental triangle ABC about point Q and noting the values of AC and BC.

The above procedure explained for the determination of the common-emitter h-parameters can also be used for determination of the common-base and common-collector h-parameters from the appropriate input and output characteristics.

10.6. VARIATIONS OF HYBRID-PARAMETERS OF A TRANSISTOR

The variation of h-parameters depends upon junction temperature, collector current and collector-to-emitter voltage V_{CE}. Among these factors, the variations due to junction temperature and collector current are significant and thus discussed here.

10.6.1. Variation of h-parameters Due to Temperature Variation. From the above discussion it is obvious that all the four h-parameters of any transistor configuration (viz. CB, CE and CC) can be determined from the slopes and spacing between curves at the quiescent point Q if it is specified. Since the characteristic curves, in general, are neither straight lines nor equally spaced for equal variations in collector-emitter voltage or base current, the values of h-parameters depend upon the position of the Q-point on the curves. It is also known that the shape and actual numerical values of the characteristic curves depend on the junction temperature. Thus h-parameters of a transistor depends upon the temperature. Most transistor specification sheets include curves of the variation of the h-parameters with the quiescent point and temperature. Usually $I_C = 1$ mA is taken as the reference collector current and collector junction temperature of 25°C is taken as the reference temperature.

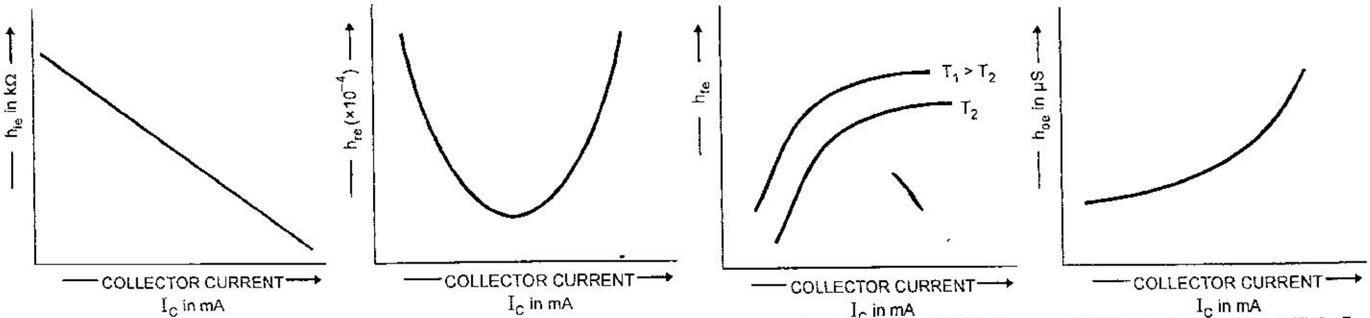
10.6.2. Variation of h-parameters Due to Variation in Collector Current

(*a*) The **parameter h_{ie}** varies with operating value of collector current I_C as shown in Fig. 10.11 (*a*) *i.e.* h_{ie} decreases with the increase in the operating value of I_C.

(*b*) The **parameter h_{re}** varies with the operating value of I_C, as depicted in Fig. 10.11 (*b*). From the curve shown in Fig. 10.11 (*b*), it is obvious that h_{re} first decreases with the increase in the operating value of I_C, reaches to a minimum value at a certain value of I_C and thereafter increases with the increase in I_C.

(*c*) The **parameter h_{fe}** varies with I_C as shown in Fig. 10.11 (*c*). The graph shown in Fig. 10.11 (*c*) indicates that h_{fe} first increase with I_C, attains a maximum value at a particular value of I_C and then decreases slightly with I_C. The value of h_{fe} is temperature dependent and increases with the rise in temperature.

(*d*) The **parameter h_{oe}** varies with I_C as indicated in Fig. 10.11 (*d*), *i.e.* h_{oe} increases with the increase in I_C but not linearly.



(a) Variation of h_{ie} With I_C (b) Variation of h_{re} With I_C (c) Variation of h_{fe} With I_C (d) Variation of h_{oe} With I_C

Fig. 10.11

10.7. TYPICAL VALUES OF h-PARAMETERS FOR A TRANSISTOR

Typical values of h-parameters of a transistor for CE, CB and CC configurations at I_E = 1.3 mA are given below in Table 10.3.

TABLE 10.3

Configuration / *Parameter*	*Common Emitter* CE	*Common Base* CB	*Common Collector* CC
h_i	1.1 kΩ	21.6 Ω	1.1 kΩ
h_r	2.5×10^{-4}	2.9×10^{-4}	$\simeq 1$
h_f	50	– 0.98	– 51
h_o	25 μS (μA/V)	0.49 μS (μA/V)	25 μS (μA/V)
$1/h_o$	40 kΩ	2.04 MΩ	40 kΩ

10.8. CONVERSION OF HYBRID PARAMETERS IN TRANSISTOR THREE CONFIGURATIONS

Some transistor manufacturers provide only the four CE hybrid parameters while others provide CB h-parameters. Sometimes, for a specific purpose, it becomes necessary to convert one set of h-parameters in one configuration to another set in another configuration. The conversion formulas can be obtained using the definitions of the parameters involved and Kirchhoff's laws.

For example, let us convert common emitter h-parameters to the common-base h-parameters. For this purpose first the CB hybrid model is drawn as shown in Fig. 10.12 (*a*) and then it is redrawn in CE configuration, as shown in Fig. 10.12 (*b*). The latter corresponds in every detail to the former except that the emitter terminal E is made common to the input and output ports.

By definitions

$$h_{re} = \left.\frac{V_{be}}{V_{ce}}\right|_{I_b=0} = \left.\frac{V_{bc} + V_{ce}}{V_{ce}}\right|_{I_b=0} = 1 + \left.\frac{V_{bc}}{V_{ce}}\right|_{I_b=0}$$

If $I_b = 0$, then $I_c = -I_e$, and the current I in h_{ob} in Fig. 10.12 (*b*) is $I = (1 + h_{fb})\, I_e = h_{ob} V_{bc}$

Applying Kirchhoff's voltage law to the output mesh of Fig. 10.12 (*b*) we have

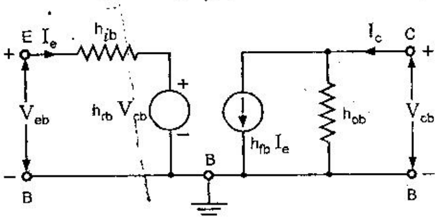
$$h_{ib} I_e + h_{rb} V_{cb} + V_{bc} + V_{ce} = 0$$

Combining the above two equations we have

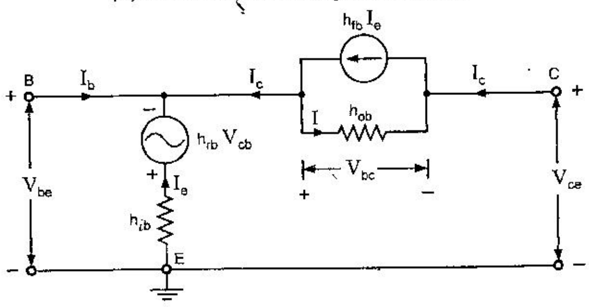
$$\frac{h_{ib} h_{ob}}{1 + h_{fb}} V_{bc} - h_{rb} V_{bc} + V_{bc} + V_{ce} = 0$$

or $$\frac{V_{bc}}{V_{ce}} = \frac{-(1 + h_{fb})}{h_{ib} h_{ob} + (1 - h_{rb})(1 + h_{fb})}$$

Thus $$h_{re} = 1 + \frac{V_{bc}}{V_{ce}} = \frac{h_{ib} h_{ob} - (1 + h_{fb}) h_{rb}}{h_{ib} h_{ob} + (1 - h_{rb})(1 + h_{fb})} \quad \text{...(10.20)}$$



(a) Common Base Hybrid Model



(b) Circuit of Fig (a) Redrawn in CE Configuration

Fig. 10.12

This is an exact expression. The simpler approximate formula is obtained by noting that, for typical values given in Table 10.3 $h_{rb} << 1$ and $h_{ob} h_{ib} << (1 + h_{fb})$

So $$h_{re} = \frac{h_{ib} h_{ob}}{1 + h_{fb}} - h_{rb} \quad \text{...(10.21)}$$

By definition $$h_{ie} = \left.\frac{V_{be}}{I_b}\right|_{V_{ce}=0}$$

If terminals C and E are connected together in Fig. 10.12 (*b*), we have circuit shown in Fig. 10.13.

From Fig. 10.13, we see that

$V_{cb} = -V_{bc} = -V_{be}$

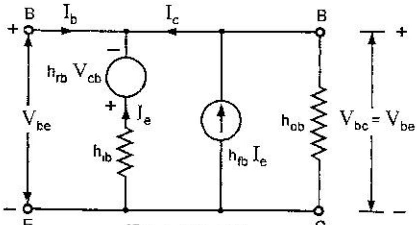


Fig. 10.13

Applying Kirchhoff's voltage law to the left hand mesh, we have

$$V_{be} + h_{ib}\,I_e + h_{rb}\,V_{cb} = 0$$

Combining the above two equations we have

$$I_e = -\frac{1-h_{rb}}{h_{ib}}\,V_{be}$$

Applying Kirchhoff's current law to node B in Fig. 10.13, we have

$$I_b + I_e + h_{fb}\,I_e - h_{ob}\,V_{be} = 0$$

$$\text{or} \quad I_b = (1 + h_{fb})\,\frac{1-h_{rb}}{h_{ib}}\,V_{be} + h_{ob}\,V_{be}$$

$$\text{Thus } h_{ie} = \frac{V_{be}}{I_b} = \frac{h_{ib}}{h_{ib}\,h_{ob} + (1-h_{rb})(1+h_{fb})} \qquad \text{...(10.22)}$$

This is an exact expression. Since $h_{rb} << 1$ and $h_{ob}\,h_{ib} << (1 + h_{fb})$, so the above equation is reduced to

$$h_{ie} \simeq \frac{h_{ib}}{1+h_{fb}} \qquad \text{...(10.23)}$$

Approximate conversion formulas for transistor parameters are given below in Table 10.4.

TABLE 10.4

Configuration / *Parameter*	*Common Emitter, CE*	*Common Base, CB*	*Common Collector, CC*	*Equivalent T-Circuit*
h_{ie}	1.1 kΩ	$\frac{h_{ib}}{1+h_{fb}}$	h_{ic}*	$r_b + \frac{r_e}{1-\alpha}$
h_{re}	2.5×10^{-4}	$\frac{h_{ib}\,h_{ob}}{1+h_{fb}} - h_{rb}$	$1 - h_{rc}$*	$\frac{r_e}{r_c\,(1-\alpha)}$
h_{fe}	50	$-\frac{h_{fb}}{1+h_{fb}}$	$-(1+h_{fc})$*	$\frac{\alpha}{1-\alpha} = \beta$
h_{oe}	25 μS (μ A/V)	$\frac{h_{ob}}{1+h_{fb}}$	h_{oc}*	$\frac{1}{r_c\,(1-\alpha)}$
h_{ib}	$\frac{h_{ie}}{1+h_{fe}}$	21.6 Ω	$-h_{ic}/h_{fc}$	$r_e + (1-\alpha)\,r_b$
h_{rb}	$\frac{h_{ie}\,h_{oe}}{1+h_{fe}} - h_{re}$	$2.9. \times 10^{-4}$	$h_{rc} - \frac{h_{ic}\,h_{oc}}{h_{fc}} - 1$	r_b/r_c
h_{fb}	$\frac{-h_{fe}}{1+h_{fe}}$	– 0.98	$-\frac{1+h_{fc}}{h_{fc}}$	$-\alpha$
h_{ob}	$\frac{h_{oe}}{1+h_{fe}}$	0.49 μS (μ A/V)	$\frac{-h_{oc}}{h_{fc}}$	$1/r_c$
h_{ic}	h_{ie}*	$\frac{h_{ib}}{1+h_{fb}}$	1.1 kΩ	$r_b + \frac{r_e}{1-\alpha}$
h_{rc}	$1 - h_{re} \simeq 1$*	1	1	$1 - \frac{r_e}{r_c\,(1-\alpha)}$
h_{fc}	$-(1+h_{fe})$*	$\frac{-1}{1+h_{fb}}$	– 51	$-\frac{1}{1-\alpha}$
h_{oc}	h_{oe}*	$\frac{h_{ob}}{1+h_{fb}}$	25 μS (μ A/V)	$\frac{1}{r_c\,(1-\alpha)}$
α	$\frac{h_{fe}}{1+h_{fe}}$	$-h_{fb}$	$\frac{1+h_{fc}}{h_{fc}}$	0.98
r_c	$\frac{1+h_{fe}}{h_{oe}}$*	$\frac{1}{h_{ob}}$	$\frac{-h_{fc}}{h_{oc}}$*	2.04 MΩ
r_e	$\frac{h_{re}}{h_{oe}}$*	$h_{ib} - \frac{h_{rb}}{h_{ob}}\,(1+h_{fb})$*	$\frac{1-h_{rc}}{h_{oc}}$*	10 Ω
r_b	$h_{ie} - \frac{h_{re}}{h_{oe}}\,(1+h_{fe})$*	$\frac{h_{rb}}{h_{ob}}$*	$h_{ic} + \frac{h_{fc}}{h_{oc}}\,(1-h_{rc})$*	590 Ω

* Stands for exact.

Example 10.3. h_{fe} = 50, h_{ie} = 0.83 kΩ, find out the current gain (h_{fb}) and input impedance (h_{ib}) for a transistor in CB configuration.

Solution :
$$h_{fb} = \frac{-h_{fe}}{1+h_{fe}} = \frac{-50}{1+50} = -0.98 \text{ Ans.}$$
$$h_{ib} = \frac{h_{ie}}{1+h_{fe}} = \frac{0.83\times10^3}{1+50} = 16.27\,\Omega \text{ Ans.}$$

Example 10.4. The h-parameters for a CE configuration are h_{ie} = 2,600 Ω, h_{fe} = 100, $h_{re} = 0.02\times10^{-2}$ and $h_{oe} = 5\times10^{-6}$ S. Find h-parameters for CC configuration.

Solution :
$$h_{ic} = h_{ie} = 2{,}600\ \Omega \text{ Ans.}$$
$$h_{fc} = -(1+h_{fe}) = -(1+100) = -101 \text{ Ans.}$$
$$h_{rc} = 1 - h_{re} \simeq 1 \text{ Ans.}$$
$$h_{oc} = h_{oe} = 5\times10^{-6}\text{ S Ans.}$$

10.9. TRANSISTOR AMPLIFIER CIRCUIT PERFORMANCE IN h-PARAMETERS

A transistor amplifier can be formed simply by connecting a signal source to the input and an external load to the output terminals of a transistor, as shown in Fig. 10.14 and giving proper bias to it. All transistor amplifiers, connected in any one of the three possible configurations, are basically two-port devices, as shown in Fig. 10.14, that is, there are a pair of input terminals and a pair of output terminals. In Fig. 10.15, the transistor has been replaced with its small-signal hybrid model without specifying the configuration. The circuit shown in Fig. 10.15 is valid for any type of load whether it be a pure resistance, an impedance or another transistor. This is true because the transistor hybrid model was derived without any regard to the external circuit in which the transistor is incorporated. For an amplifier there are six quantities of great interest (input impedance, output impedance, current gain, voltage gain, power gain and phase relationship), each of which will be discussed in detail here.

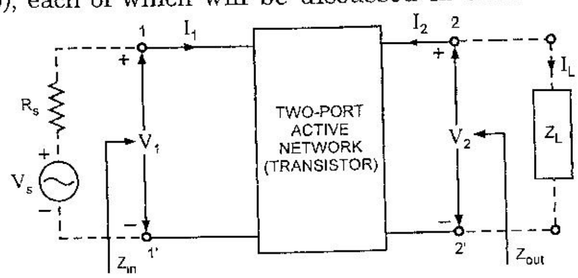


Fig. 10.14. *Basic Amplifier Circuit*

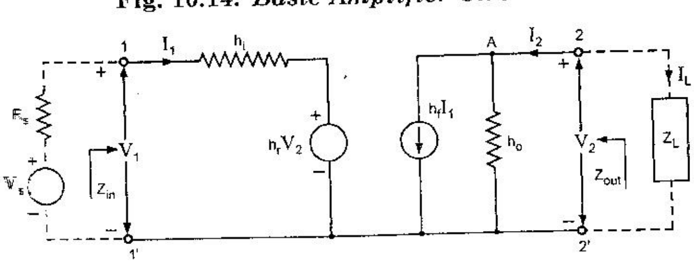


Fig. 10.15. *Small Signal Hybrid Model of Transistor Amplifier Circuit Shown in Fig. 10.14*

For the resulting equations to be useful, however, the quiescent point must be established and the resulting h-parameters must be known :

1. *Input Impedance.* The input impedance Z_{in} is defined as the ratio of input voltage to the input current *i.e.*
$$Z_{in} = \frac{V_1}{I_1} \qquad ...(10.24)$$
From the input circuit shown in Fig. 10.15 we have
$$V_1 = h_i I_1 + h_r V_2 \qquad ...(10.25)$$
Substituting $V_1 = h_i I_1 + h_r V_2$ in Eq. (10.24) we have
$$Z_{in} = \frac{h_i I_1 + h_r V_2}{I_1} = h_i + h_r \frac{V_2}{I_1} \qquad ...(10.26)$$
From the output circuit shown in Fig. 10.15 we have
$$I_2 = h_f I_1 + h_o V_2 = \frac{-V_2}{Z_L} \qquad ...(10.27)$$
The minus sign is used here because the actual current flows in the direction opposite to that of I_2.
$$\text{So } \frac{V_2}{I_1} = \frac{-h_f}{h_o + \frac{1}{Z_L}} \qquad ...(10.28)$$
Substituting value of $\frac{V_2}{I_1}$ from Eq. (10.28) in Eq. (10.26) we have
$$\text{Input impedance } Z_{in} = h_i - \frac{h_r h_f}{h_o + \frac{1}{Z_L}} \qquad ...(10.29)$$
$$\simeq h_i \qquad \text{...if } h_r \text{ or } Z_L \text{ is very small.}$$
It is seen that Z_{in} depends on Z_L *i.e.* ac resistance of the load across the output terminals of the transistors.

2. *Current Gain.* The current gain is denoted by A_i and is defined as the ratio of output current to input current.
$$i.e.\ A_i = \frac{I_L}{I_1} = \frac{-I_2}{I_1} \qquad ...(10.30)$$
Applying Kirchhoff's current law to node A in the output circuit
$$I_2 = h_f I_1 + h_o V_2$$
Substituting $V_2 = -I_2 Z_L$ from Eq. (10.27) we have
$$I_2 = h_f I_1 - h_o I_2 Z_L$$
$$\text{or Current gain } A_i = \frac{-I_2}{I_1} = \frac{-h_f}{1+h_o Z_L} \simeq -h_f \text{ if } Z_L \text{ is zero or } h_o Z_L << 1 \qquad ...(10.31)$$

Current Gain Taking R_s into Account. The source current is not the transistor input current because it partly flows through R_s and partly through Z_{in}. So the voltage source V_s with series source resistance R_s is replaced by the Norton's equivalent source, shown in Fig. 10.16, consisting of current source I_s with source resistance R_s in shunt. The overall current gain A_{is} is given as

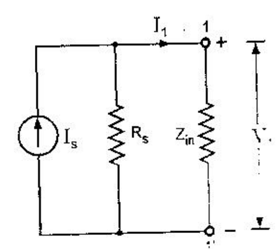


Fig. 10.16. *Norton's Equivalent For The Source*

$$A_{is} = \frac{-I_2}{I_s} = \frac{-I_2}{I_1} \cdot \frac{I_1}{I_s} = A_i \frac{I_1}{I_s} \quad \ldots(10.32)$$

From Fig. 10.16, $\frac{I_1}{I_s} = \frac{R_s}{Z_{in} + R_s}$

$$\text{So } A_{is} = \frac{A_i R_s}{Z_{in} + R_s} \quad \ldots(10.33)$$

Note that if $R_s = \infty$ then $A_{is} = A_i$. Hence A_i is the *current gain for an ideal current source* (one with infinite source resistance).

3. ***Voltage Gain.*** The voltage gain is denoted by A_v and is defined as the ratio of output voltage to the input voltage *i.e.*

$$A_v = \frac{V_2}{V_1} = \frac{V_2}{I_1 Z_{in}} = \frac{-h_f}{\left(h_o + \frac{1}{Z_L}\right) Z_{in}} \quad \ldots(10.34)$$

$$\because \text{ from Eq. (10.28) } \frac{V_2}{I_1} = \frac{-h_f}{h_o + \frac{1}{Z_L}}$$

Voltage Gain Taking R_s into Account. Overall voltage gain A_{vs} is given as

$$A_{vs} = \frac{V_2}{V_s} = \frac{V_2}{V_1} \cdot \frac{V_1}{V_s} = A_v \cdot \frac{V_1}{V_s} \quad \ldots(10.35)$$

From the equivalent input circuit of the amplifier given in Fig. 10.17, we have

$$\frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in} + R_s},$$

$$\text{So } A_{vs} = A_v \frac{Z_{in}}{Z_{in} + R_s} \quad \ldots(10.36)$$

Note that if $R_s = 0$ the $A_{vs} = A_v$. Hence A_v is the voltage gain for an ideal voltage source (one with zero internal resistance). In practice, the quantity A_{vs} is more meaningful than A_v since usually, the source resistance has significant effect on the overall voltage amplification.

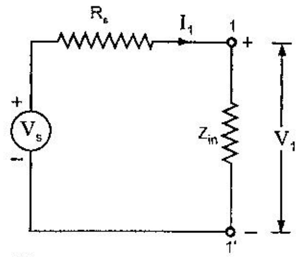


Fig. 10.17. ***Thevenin's Equivalent For The Source***

Independent of the transistor characteristics, the voltage and current gains taking source impedance into account, is related as

$$A_{vs} = A_{is} \frac{Z_L}{R_s} \quad \ldots(10.37)$$

provided that the current and voltage generators have the same source resistance R_s.

4. ***Output Admittance.*** The output admittance Y_{out} is defined as the ratio of the output current to output voltage with $V_s = 0$ *i.e.*

$$Y_{out} = \frac{I_2}{V_2} \text{ with } V_s = 0 \quad \ldots(10.38)$$

$$= h_f \frac{I_1}{V_2} + h_o \quad \ldots(10.39)$$

$$\because \text{ from Eq. (10.27), } I_2 = h_f I_1 + h_o V_2$$

Now from input circuit shown in Fig. 10.15 we have $R_s I_1 + h_i I_1 + h_r V_2 = 0$ ...(10.40) taking $V_s = 0$

$$\text{or } \frac{I_1}{V_2} = -\frac{h_r}{h_i + R_s} \quad \ldots(10.41)$$

Substituting $\frac{I_1}{V_2} = \frac{-h_r}{h_i + R_s}$ from Eq. (10.41) in Eq. (10.39) we have

$$Y_{out} = h_o - \frac{h_f h_r}{h_i + R_s} \quad \ldots(10.42)$$

It is to be noted that *output admittance is a function of source resistance* R_s. If the source impedance is purely resistive, as it has been assumed, then Y_{out} is real (purely conductance).

In the above definition of Y_{out}, the load Z_L has been considered external to the amplifier. If the output impedance of the amplifier stage with Z_L included is required, the load impedance can be determined as the parallel combination of Z_L and Z_{out} (*i.e.*, $Z_L \parallel Z_{out}$).

5. ***Power Gain.*** The average power delivered to the load shown in Fig. 10.14, P_2 equals $|V_2|\,|I_L|\cos\phi$ where ϕ is the phase angle between V_2 and I_L. For situation under discussion power delivered, $P_2 = -V_2 I_2 \cos\phi$. The minus sign arises for the same reason discussed in the derivation of input impedance. It indicates that the load is absorbing power and not supplying to the amplifier circuit. For purely resistive load, $\cos\phi = 1$ and $P_2 = -V_2 I_2$. The input power is $V_1 I_1$. So power gain

$$A_P = \frac{P_2}{P_1} = \frac{-V_2}{V_1} \cdot \frac{I_2}{I_1} = -A_v A_i \quad \ldots(10.43)$$

$$\text{As } V_2 = -I_2 R_L \text{ and } I_2 = A_i I_1$$

$$\therefore A_v = \frac{V_2}{V_1} = \frac{-A_i I_1 R_L}{V_1} = \frac{-A_i R_L}{V_1/I_1} = \frac{-A_i R_L}{Z_{in}}$$

$$\text{So power gain, } A_p = -A_v A_i = -\left(\frac{-A_i R_L}{Z_{in}}\right) A_i = \frac{A_i^2 R_L}{Z_{in}} \quad \ldots(10.44)$$

6. ***Phase Relationship.*** The phase relationship between the output current or voltage and the input current and voltage can be determined simply examining the Eqs. (10.30), (10.31) and (10.34).

All the h-parameters are positive except h_f for the common-base and common-collector configurations (Table 10.3). For the common-emitter configuration it is obvious from Eqs. (10.30), (10.31) and (10.34) that for the purely resistive loads, the transistor output current I_2 is in phase with the input current I_1 while the output voltage, due to the negative sign, has a polarity opposite to that of the input voltage *i.e.* output voltage attains its negative maximum when the input signal attains its positive maximum. For the common-collector and common-base configurations the opposite is true because of negative value of h_f.